

# A new introductory quantum mechanics curriculum

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**IOP** Institute of Physics

<http://quantumphysics.iop.org>

<http://arxiv.org/abs/1307.1484>

[www.compadre.org/per/perc/2013/Detail.cfm?id=5225](http://www.compadre.org/per/perc/2013/Detail.cfm?id=5225)

2013 AAPT Summer Meeting, July 13 - 17, Portland, Oregon



University of  
St Andrews

600  
YEARS

# Why a new curriculum?

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Review of approaches in the UK, IOP workshop  
December 2011 (led by Derek Raine)

Aims of the New QC project:

Provide freely available online material for the  
introductory level developing the theory using  
two-level systems.

- Focus on experiments that have no classical explanation, interpretive aspects of quantum mechanics and quantum information theory
- Mathematically less challenging (much of the linear algebra needed part of the resource)

# The New QC Team

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- Pieter Kok, Dan Browne, Mark Everitt  
*Authors of ~80 short(ish) articles centred on questions.*
- Antje Kohnle  
*17 Interactive simulations and accompanying activities (coding by Inna Bozhinova, Aleksejs Fomins, Gytis Kulaitis, Martynas Prokopas)*
- Derek Raine (*academic editor*), Elizabeth Swinbank (*copy editor*)
- Christina Walker (IOP)  
*Project management*

# Overview of the content (Quantum information theme)

- “Photons first”: Single photon interference via the Mach-Zehnder interferometer
- Spin  $\frac{1}{2}$  particle in a Stern-Gerlach apparatus
- Two-level atoms
- Two-particle states, entanglement, Bell inequality, teleportation, no cloning theorem, quantum key distribution, quantum computing
- Transition to continuous systems

# Website structure

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- Five different themes (informational, foundational mathematical, physical, historical)
- Navigation panel with related content (prerequisites, further reading, glossary, exercises, sims); Users can rate difficulty, navigation panel shows all articles read with ratings.
- All content free to use, downloadable; full download and solutions available to instructors. Instructor resources (exemplars of use, forum). In future, IOP aims to provide instructors with information on student use and ratings.

# Overview of the simulations

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- Aims: enhance engagement, exploration, linking of multiple representations
- Make use of principles of interface design from other studies (PhET, Physlets, Clark&Mayer, etc.)
- Depict simplified, idealized situations, make the invisible visible; simple startup configuration
- Include text explanations; self-contained instructional tools
- Accompanying activities aim to promote guided exploration and sense-making; solutions available to instructors
- Topics: Linear algebra, fundamental physics concepts, single photon interference, Bloch sphere, entanglement, hidden variables, quantum information.

## Graphical representation of complex eigenvectors

### Transformation matrix

$\hat{O}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\hat{O}_2 = \begin{pmatrix} 0 & e^{-0.25i\pi} \\ e^{0.25i\pi} & 0 \end{pmatrix}$

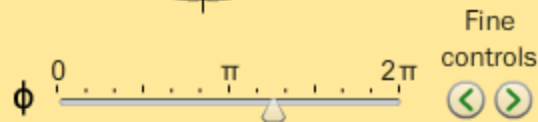
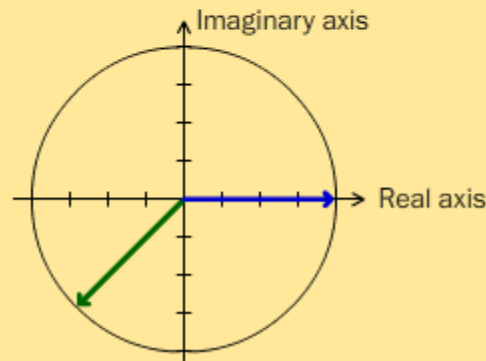
$\hat{O}_3 = \begin{pmatrix} 0 & e^{-0.5i\pi} \\ e^{0.5i\pi} & 0 \end{pmatrix}$

$\hat{O}_4 = ?$  Find the matrix elements!

Display help on exponential form

×

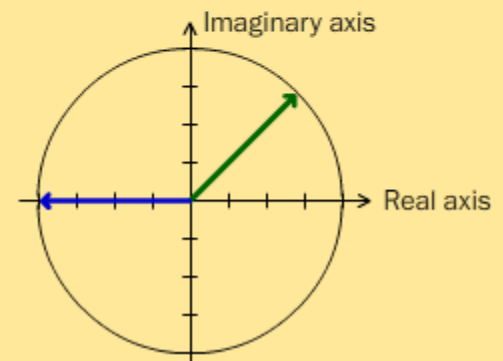
### Initial vector components



$$\vec{n} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\phi} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{1.250 i\pi} \end{pmatrix}$$

=

### Transformed vector components



Eigenvector: **YES** Eigenvalue: -1

$$\hat{O} \cdot \vec{n} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{1.000 i\pi} \\ e^{0.250 i\pi} \end{pmatrix} = -1 \vec{n}$$

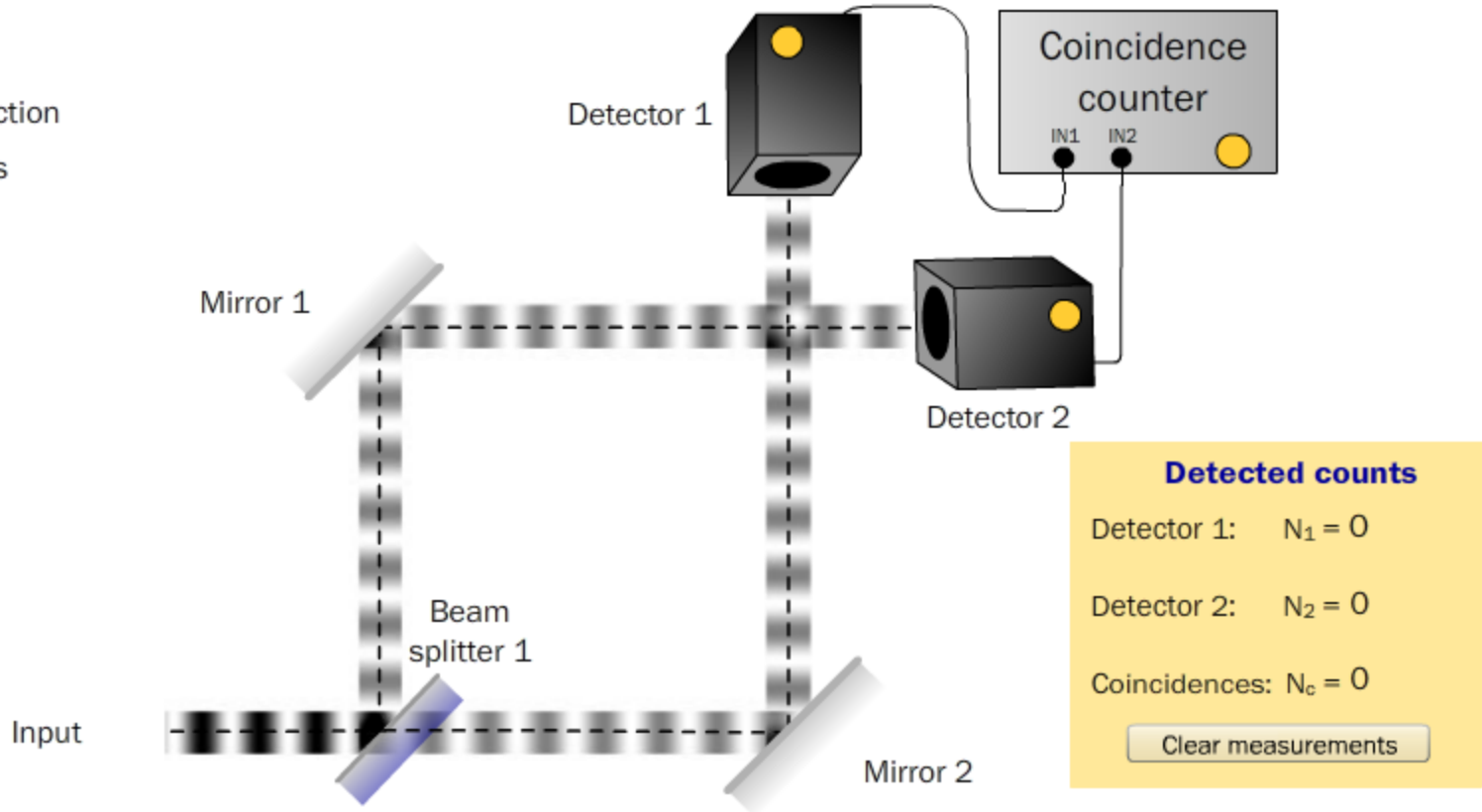
The graphs show the components (component 1 in blue, component 2 in green) of a two-dimensional complex unit vector  $\vec{n}$  in the complex plane and the components of the transformed vector  $\hat{O}\vec{n}$ , where  $\hat{O}$  is a  $2 \times 2$  complex matrix that transforms the unit vector into a new vector in the complex plane. The first component of the vector is taken (as is often the convention in quantum mechanics) to be real and positive, and here of magnitude  $1/\sqrt{2}$ . The second component however is complex.

Use the slider for the angle  $\phi$  to change the direction of the second component of the vector  $\vec{n}$  in the complex plane, and the buttons to choose different transformation matrices. Complex numbers are displayed in the exponential form ( $re^{i\theta}$ ) with modulus  $r$  and argument  $\theta$ . In quantum mechanics, such a transformation matrix would represent an operator, and the vector would represent a quantum state. Note that the radius of the circle is  $1/\sqrt{2}$ .



## Interferometer experiments with photons, particles and waves

- Introduction
- Controls



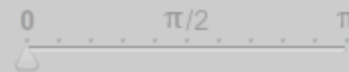
### Input

- Classical particles
- Electromagnetic wave
- Single photons

### Main Controls

- Not available for waves!
- Stop
- Fast forward 50 counts
- Insert second beam splitter

### Phase shift in lower path



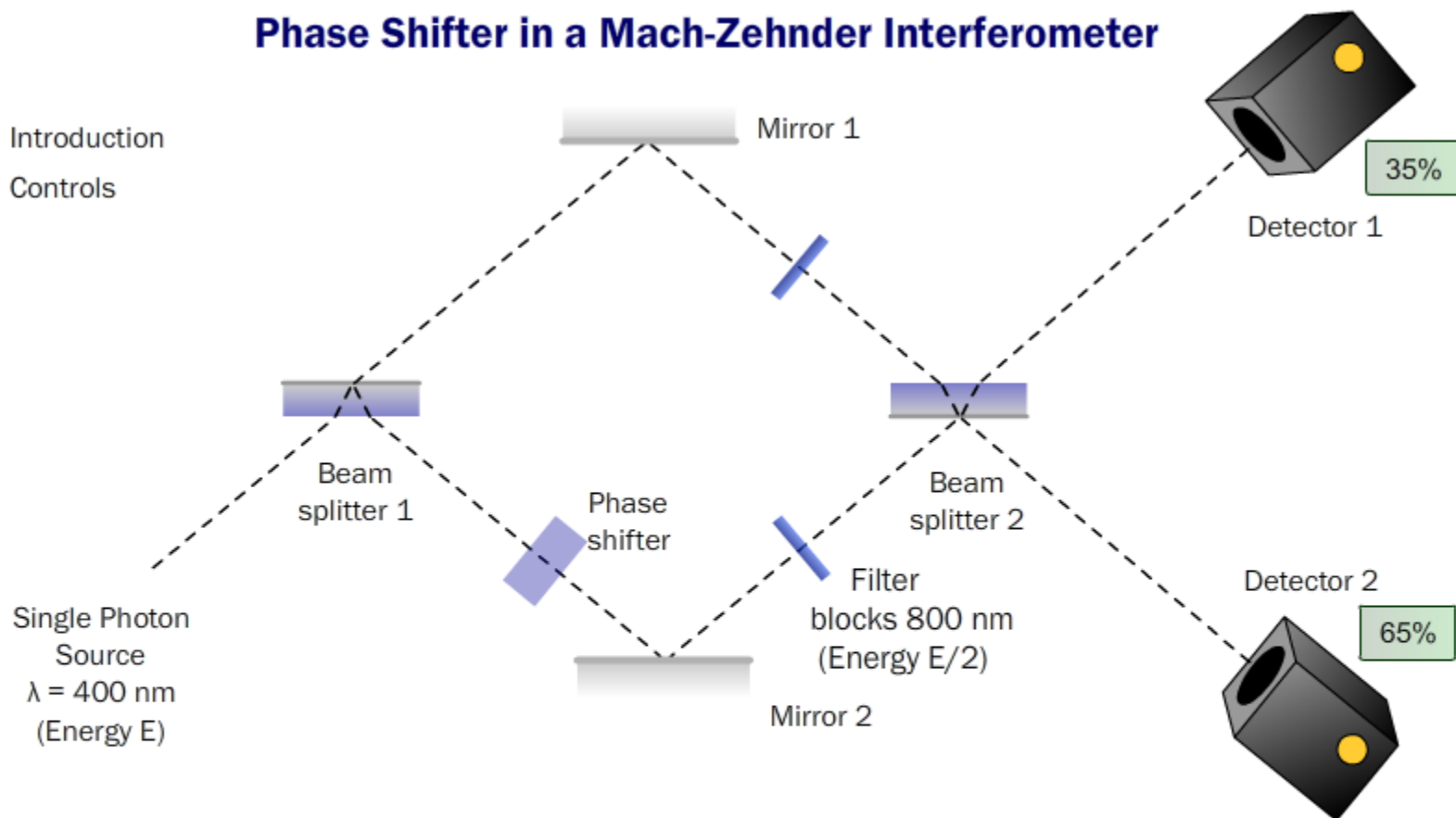
### Display controls

- Label elements
- Show theoretical intensities (Input intensity is 100%)



## Phase Shifter in a Mach-Zehnder Interferometer

- Introduction
- Controls

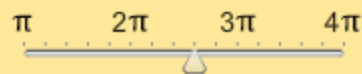


### Main Controls





### Phase Shift $\phi$



### Display Controls

- Label optical elements
- Show quantum states
- Show theoretical probabilities
- Show matrix representations of optical elements

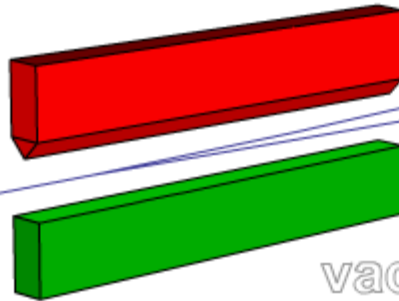


## The expectation value of an operator

Input state:

$\frac{1}{\sqrt{5}}(2|\uparrow\rangle + |\downarrow\rangle)$

$a|\uparrow\rangle + b|\downarrow\rangle$   
Find a and b!



vacuum

Show introduction



### Number of measurements

Total measurements:  $N_{\text{tot}} = 54$

Outcome  $S_z = +\frac{\hbar}{2}$ :  $N_+ = 42$

Outcome  $S_z = -\frac{\hbar}{2}$ :  $N_- = 12$

Clear measurements

### Main controls

Send spin 1/2 particles through the Stern-Gerlach apparatus

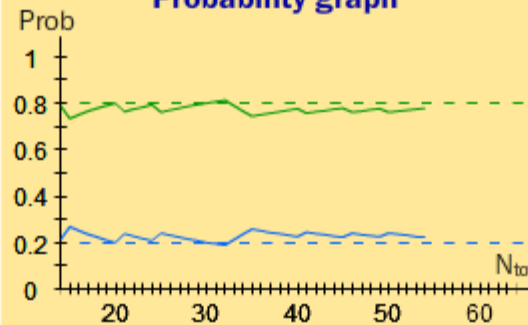
Take more measurements

Single particle

Continuous stream of particles

Fast forward 50 particles

### Probability graph



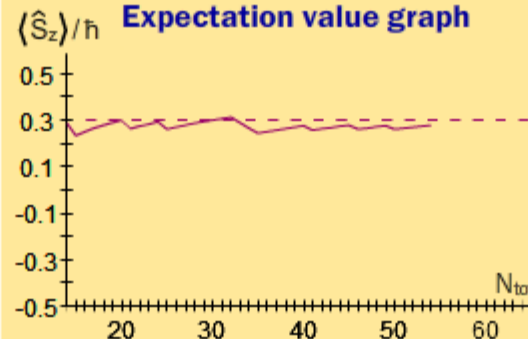
### Probabilities

	Observed	Theoretical
$S_z = +\frac{\hbar}{2}$ : $\text{Prob}_+ = \frac{N_+}{N_{\text{tot}}} = 0.778$	0.778	0.8
$S_z = -\frac{\hbar}{2}$ : $\text{Prob}_- = \frac{N_-}{N_{\text{tot}}} = 0.222$	0.222	0.2

### Display controls

- Show probabilities
- Show probability graph
- Show expectation value
- Show expectation value graph

### Expectation value graph

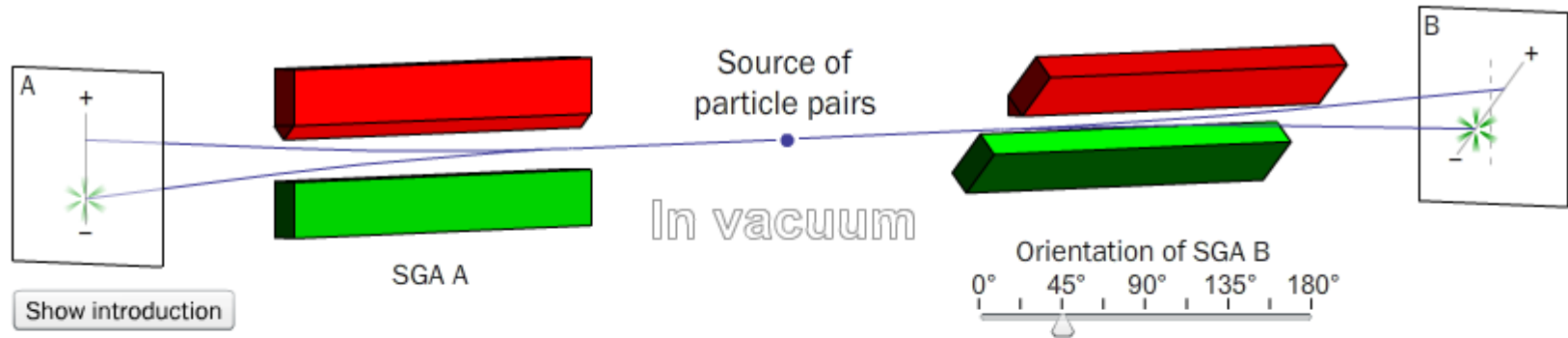


### Expectation value

	Mean of measurement outcomes	Theoretical
$\langle \hat{S}_z \rangle = (+\frac{\hbar}{2})\text{Prob}_+ + (-\frac{\hbar}{2})\text{Prob}_-$	0.278 $\hbar$	0.3 $\hbar$



## Entangled spin $\frac{1}{2}$ particle pairs versus an elementary hidden variable theory



### Main controls

Quantum theory

Hidden variable theory

Send pairs with pre-determined spin vectors through the SGAs.

Single particle pair

Continuous stream of pairs

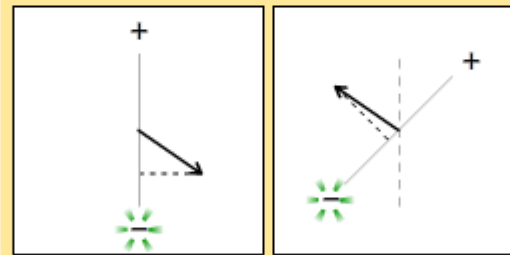
Fast forward 50 particle pairs

### Hidden variables

Locally pre-determined opposite spin vectors

A

B



### Paired Measurements

$N_{\text{total}} = 156$  | Probabilities | Quantum prediction

Outcome Same (A,B) = (+,+) or (-,-):

$N_{\text{same}} = 40$  |  $P_{\text{same}} = \frac{N_{\text{same}}}{N_{\text{total}}} = 0.256$  | 0.146

Outcome Opposite (A,B) = (+,-) or (-,+):

$N_{\text{opp}} = 116$  |  $P_{\text{opp}} = \frac{N_{\text{opp}}}{N_{\text{total}}} = 0.744$  | 0.854

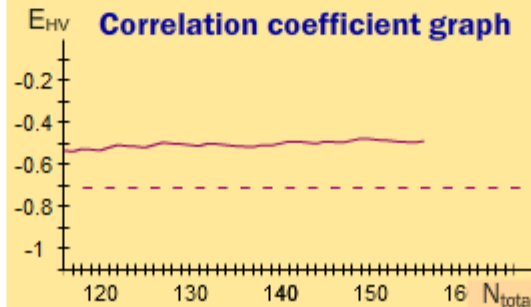
Clear measurements

### Display controls

Show correlation coefficient

Show correlation coefficient graph

### Correlation coefficient graph

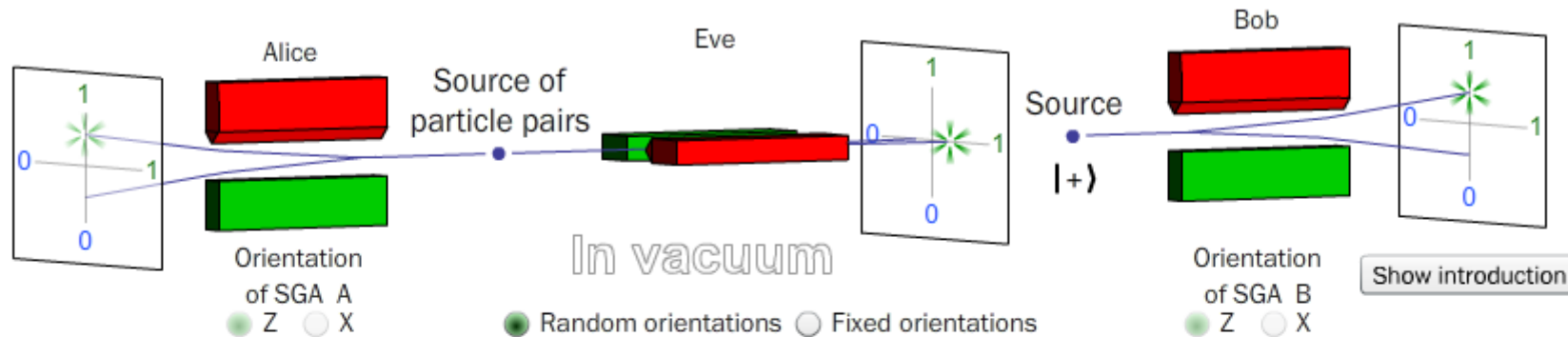


### Correlation coefficient

Expectation value E: average of measurement outcomes AB, where AB = +1 (same), AB = -1 (opposite) | Quantum prediction

$E_{\text{HV}}(AB) = P_{\text{same}} - P_{\text{opp}} = -0.488$  | -0.708

## Quantum key distribution with entangled spin 1/2 particles



### Main controls

Send entangled spin 1/2 particle pairs

Take more measurements

Single particle pair

Continuous stream of particle pairs

Fast forward 50 particle pairs

Let Eve intercept and resend particles

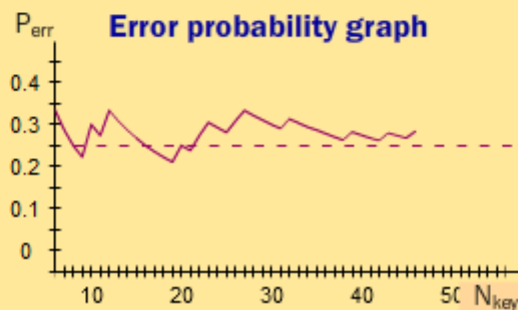
Stop eavesdropping

Alice		Eve		Bob		Alice and Bob	Key
Basis	Outcome	Basis	Outcome	Basis	Outcome	Same bases?	Bob inverts value
Z	1	X	1	Z	1	YES	ERROR
Z	1	Z	0	Z	0	YES	1
Z	0	Z	1	Z	1	YES	0
X	0	Z	1	X	0	YES	ERROR
X	1	X	0	Z	0	NO	
X	0	Z	0	X	1	YES	0
X	1	X	0	X	0	YES	1

### Display controls

- Show secure key
- Show errors
- Show error probability graph

Clear measurements



Errors		Theoretical
Total pairs:	$N_{tot} = 100$	
Key bits:	$N_{key} = 46$	$0.5 N_{tot}$
Errors:	$N_{err} = 13$	$0.25 N_{key}$
Probability:	$P_{err} = \frac{N_{err}}{N_{key}} = 0.283$	0.25



# Optimization of simulations and activities

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- See PERC poster (A Kohnle, C Baily, C Hooley, B Torrance)
- 38 hours of observation sessions with 17 student volunteers in February/May 2013 from the St Andrews introductory level.
- Use of two simulations (single photon interference, hidden variables) in the Boulder modern physics course (N Finkelstein, C Baily)
- Use of three simulations (+ entanglement) in the St Andrews quantum physics course.
- Revisions to all simulations and activities where appropriate

# Future activities

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- Multi-institutional observation sessions and trials in introductory qm courses.
- Build a community of users (instructor resources, exemplars of use, user forum).
- Further sims (+ all as HTML5/JS touchscreen versions) and additional activities (more exploratory and collaborative in nature); revisions to articles based on user input.
- Investigate student difficulties with the New QC. Develop research-based interactive engagement materials / concept tests etc.